THEORY OF A COARSE-DISPERSED FLUIDIZED BED

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A monodispersed system of large spheres in a gas, where momentum and energy exchange between the particles results mainly from their direct collisions, is studied. Mass and momentum conservation equations for both phases and a transfer equation for pulsatory energy of the particles are given, and all rheological relations, closing these equations, are derived.

A theoretical analysis of flows of dispersed systems with large particles and, in particular, of coarse-dispersed fluidized beds, as well as an investigation of transfer processes therein are complicated because until now there has been no closed system of appropriate hydrodynamic equations. Basic difficulties in formulating the conservation equations and the rheological relations closing them are associated with the development of a fairly high-rate pulsatory motion in such systems, which has a considerable, and sometimes even determining impact both on rheological properties of the phases and on effective transfer coefficients. Therefore, various versions of the semiempirical equations involving a priori unknown functions and parameters (see representative examples in [1-6]) are generally utilized in practice.

A constructive solution of the stated problem requires, first and foremost, the construction of a physically acceptable model of pulsatory motion and the identification of the principal mechanism of pulsation generation. It was suggested in [7] to regard energy transfer to the pulsations from an average relative flow of the liquid phase due to the work of the latter on random concentration fluctuations of the dispersed system as such a mechanism. Since the force of hydrodynamic interaction between the particles and the carrier medium is nonlinear by a local concentration, the balance of forces acting on a certain particle is randomly upset, as a result of which the particle is involved in the pulsatory motion. Later on, a number of works, particularly [8], applied the model of such "pseudoturbulent" motion to analyzing the pulsation properties with success.

Studies [9, 10] assumed that the particle pulsations arise from the action of random Magnus forces. In principle, such a viewpoint could be correct provided the particles rotate at fairly high random angular velocities. Forasmuch as the rotation of the particle comes out of the transfer with a part of the translational energy of the particles at their collisions, in real situations the energy of any rotational degree of freedom cannot, on the average, exceed the energy of one translational degree of freedom for the particle. It appears here [11] that, even under a hardly realistic assumption that the pulsatory energy distributions in translational and rotational degrees of freedom are equal, a contribution of the Magnus forces to the pulsation generation even for large particles proves relatively insignificant.

Average pseudoturbulence characteristics for both phases of the dispersed system are strongly dependent on the physical mechanism of momentum and energy exchange between the particles. In suspensions of rather small particles in droplet liquids, this exchange is effected through random velocity and pressure fields in the liquid, whereas the role played in this process by direct collisions is negligible (exactly the same situation was treated, in particular, in [6, 8]). The second limiting case corresponds to suspensions of large particles in gases when, on the contrary, interparticle collisions are dominant in the exchange (see [9-11] and [12, 13]). In the first case, when analyzing the random behavior of the particles, it is natural to employ a kinetic equation of Fokker-Planck type, incorporating diffusion in the velocity space, caused by the particle interaction with the liquid phase pulsations [8]. In the second case, it is not less natural to use the Boussinesq equation and the Enskog method for its solution under weak nonequilibrium conditions. In the first case, taking account of new terms, stipulated by pseudoturbulence in the momentum conservation equations, enables the construction of a hydromechanical model for motion of fine-dispersed suspensions, devoid of a number of fundamental drawbacks inherent in previously known models [14, 15]. In the second case, a complete solution of all arising problems, and an adequate consideration of pulsations in the system of hydrodynamic equations for the average motion have failed until now, notwithstanding the formalized attempts undertaken in [12, 13].

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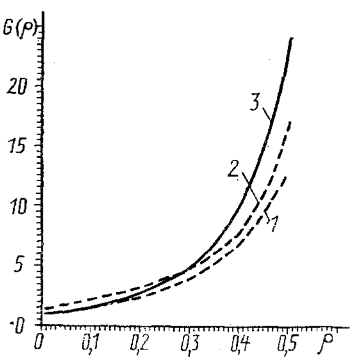

Fig. 1

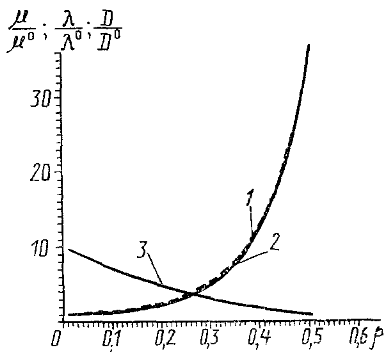

Fig. 2

Fig. 1. Function G(p) from Eqs. (7), (8), and (9) (dashed (1, 2) and solid (3) curves, respectively).

Fig. 2. Dimensionless transfer coefficients μ/μ^0 (curve 1), λ/λ^0 (2), and D/D^0 (3).

The current study constructively solves the above-stated problem for a gas suspension of identical large spherical particles. As long as the momentum and energy exchange between the particles results chiefly from direct collisions, the principle of pulsatory energy equidistribution in the translational degrees of freedom of the particles appears to be approximately valid. For the sake of simplicity, we totally neglect the energy of rotational degrees of freedom, which allows a disregard of the appropriate Magnus forces as well. In this respect, the situation under consideration is similar to that in statistical mechanics of polyatomic gases with temperatures which are not too high, when the rotational (as well as oscillatory) degrees of freedom of the molecules have not actually been excited. A suitable generalization of the theory can, in principle, be executed further on using, for example, the Eucken method as well as the reasons concerning the allowance for the Magnus forces, outlined in [11].

Ignoring the particle interaction with the carrier gas and a possible energy loss at collisions, an evolution of the system of randomly pulsating particles would be governed by the ordinary Boltzmann equation, wherein the role of the temperature kT in energy units, which characterizes the state of the molecular gas, is played by a doubled energy of the pulsatory particle motion per translational degree of freedom, i.e., by the quantity $\theta = m\langle\omega'^2\rangle/3$. In this case, the variation rates for average density, momentum, and pulsatory energy of the particle gas, caused by collisions, may be represented in standard form, which also permits us to write the relevant conservation equations [16, 17]. The forces affecting the particles from the side of the surrounding gas, the energy supply from the average carrier flow to the pulsations, and the pulsatory energy dissipation by viscous stresses in the gas and by resilience at collisions can afterwards be taken into account via introducing appropriate relatively small systems into these equations. Confining ourselves to the ordinary Navier–Stokes approximation, based on [16] we can write the conservation equations:

for particle mass

$$\frac{\partial\rho}{\partial t} + \nabla(\rho\mathbf{w}) = 0, \quad \rho = \sigma n, \tag{1}$$

for momentum

$$\rho d_1\left(\frac{\partial}{\partial t} + \mathbf{w}\nabla\right)\mathbf{w} = \nabla\mathbf{P} + n\langle\mathbf{f}\rangle + \rho d_1 g \tag{2}$$

and for pulsatory energy

$$\left(\frac{\partial}{\partial t} + \mathbf{w}\nabla\right)\theta = \frac{2}{3n}[\mathbf{P}:(\nabla*\mathbf{w}) - \nabla\mathbf{Q} + q - q_v - q_c]. \tag{3}$$

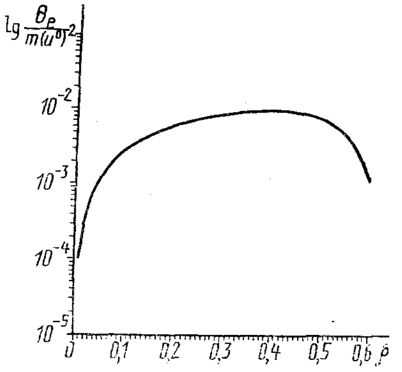

Fig. 3

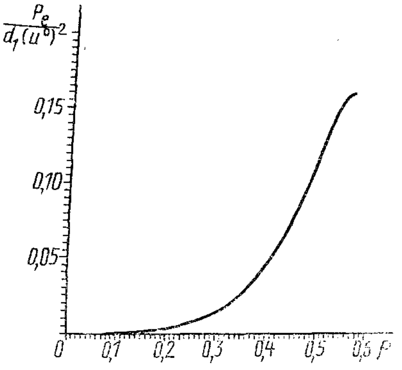

Fig. 4

Fig. 3. Dimensionless effective temperature of the particle gas in the equilibrium state.

Fig. 4. Dimensionless equilibrium pressure of the particle gas.

Here, we have introduced a stress tensor in the particle system

$$\mathbf{P} = -P\mathbf{I} + 2\mu\,[\mathbf{E}_w - (\mathrm{Sp}\mathbf{E}_w/3)\,\mathbf{I}] \tag{4}$$

and a pulsatory energy flux

$$\mathbf{Q} = -\lambda\nabla\theta, \tag{5}$$

The asterisk marks an operation of dyadic multiplication of vectors, the colon denotes convolution for second-rank tensors by both indexes, and $\mathbf{E}_w$ represents the deformation rate tensor constructed for the field of average particle velocity $\mathbf{w}(t,\mathbf{r})$. The problem lies in completely identifying the quantities entering into Eqs. (4) and (5), and in finding the average force $n\langle\mathbf{f}\rangle$ of interphase interaction in Eq. (2), and the quantities q, q_v, and q_c, which describe, correspondingly, the energy supply from the carrier flow to the pulsations, the viscous dissipation in the gas, and the energy dissipation due to collision inelasticity in a computation per unit volume of the mixture and per unit time.

An equation of state for a concentrated system of rigid spheres can be represented as

$$P = G(\rho)\,n\theta, \quad G(\rho) = 1 + 4\rho\chi(\rho), \tag{6}$$

where the Enskog factor $\chi(\rho)$ defines the number of times by which the frequency of collisions in the indicated system differs from that in the gas of point particles, with the same numerical concentration n [17]. The use of the equations of state for determining the pressure of the particle gas in Eq. (4) implicates, as usual, a hypothesis about local thermodynamic equilibrium of a nonequilibrium system.

The function $G(\rho)$ in Eq. (6) may be obtained traditionally with the aid of virial expansions. Here we employ a different method. Away from a close-packing state of the spherical particles, it is reasonable to utilize a simple analytic result corresponding to the Carnahan–Starling version [18] of the familiar Percus–Yevik approximate theory of dense gases and liquids. Then

$$G(\rho) = \frac{1 + \rho + \rho^2 - \rho^3}{(1-\rho)^3}, \quad \chi(\rho) = \frac{1 - \rho/2}{(1-\rho)^3}. \tag{7}$$

Near the close-packing state, it is reasonable to avail ourselves of a smoothed free space model of the Enskog theory of dense gases, according to which [17]:

$$G(\rho)=\frac{1}{1-(\rho/\rho_*)^{1/3}}, \quad \chi(\rho)=\frac{(\rho/\rho_*)^{1/3}}{4\rho\,[1-(\rho/\rho_*)^{1/3}]}, \tag{8}$$

where ρ_* is the volume concentration of the particles, associated with the close-packing state. Since there is certain indefiniteness and ambiguity as to the structure of the close-packing state, we regard ρ_* as a semiempirical parameter below; in calculations given further we assume $\rho_* = 0.6$. The equation of state of the form (6) with the Enskog factor from Eq. (7) was previously used with success to analyze fine-dispersed suspensions and colloids in [14, 15, 19]. The relations (8) were employed in the theory of a fluidized bed in [7].

Instead of Eqs. (7) and (8), it is desirable to derive general formulae, approximately valid throughout the concentration interval (D, ρ_*). Ogawa et al. [20] suggested using merely Eqs. (8) as such formulae. The latter, however, are obviously unfit for $\rho < (0.10\text{-}0.15)\rho$ [17]. Therefore, below we utilize the interpolation formula

$$G(\rho)=\frac{1+\rho+\rho^2-\rho^3}{(1-\rho)^3}+\frac{1}{1-(\rho/\rho_*)^{1/3}}-\sum_{j=0}^{J}\left(\frac{\rho}{\rho_*}\right)^{j/3}, \tag{9}$$

obtained by analogy with the known procedure of matching asymptotic expansions. That is, an expansion of the second formula for small ρ is subtracted from the sum of Eqs. (7) and (8), which allows a correction of the anomalous behavior of Eq. (8) in the region of low concentrations. Should $J = 6$ be taken in Eq. (9), a discrepancy between Eqs. (9) and (7) in this region will be of the order of $\rho^{7/3} < \rho^2$. The interpolation character in Eq. (9) is clear from Fig. 1.

In conformity with the Enskog theory [17], the following expressions are valid for the coefficients entering into Eqs. (4) and (5)

$$\begin{aligned}\mu &= 4\rho\,(y^{-1}+0{,}8+0{,}76y)\,\mu^\circ,\\ \lambda &= 4\rho\,(y^{-1}+1{,}2+0{,}75y)\,\lambda^\circ,\\ y &= G(\rho)-1,\end{aligned} \tag{10}$$

where the degree sign above denotes the viscosity and the thermal conductivity (in the given case, the transfer coefficient of pulsatory energy) of a rarefied gas [16,17]:

$$\mu^\circ=\frac{5}{64a^2}\left(\frac{m\theta}{\pi}\right)^{1/2}, \quad \lambda^\circ=\frac{15}{4}\,\frac{\mu^\circ}{m}=\frac{75}{256a^2}\left(\frac{\theta}{\pi m}\right)^{1/2}. \tag{11}$$

Equations (6) and (9)-(11) fully define all rheological parameters appearing in representations for the stress tensor (4) and for the pulsatory energy flux (5) as functions of the unknown variables ρ and θ of Eqs. (1)-(3).

An evaluation of the effective coefficients of heat and mass transfer in fluidized beds and in other coarse-dispersed mixtures usually requires a self-diffusion coefficient of the pulsating particles. In the framework of the already utilized theory of dense gases, we have

$$D=\frac{4\rho}{y}\,D^\circ, \quad D^\circ=\frac{3}{32a^2n}\left(\frac{\theta}{\pi m}\right)^{1/2}. \tag{12}$$

Figure 2 plots the dimensionless coefficients of viscosity, pulsatory energy transfer, and particle self-diffusion vs the volume concentration for the case when y is calculated from Eq. (9). The first two are characterized by minima attained at the values of ρ corresponding to $y = 1.146$ and 1.151, respectively. On transition to the close-packing state, μ and λ tend to infinity, whereas D tends to zero.

The equations of mass and momentum conservation for a liquid state can be employed in Eulerian form because the characteristic linear scale of the average flow is assumed to be much larger than the particle radius a, so that average velocity gradients in the gaps between the particles v are much smaller than local velocity gradients [9, 10]. Neglecting also inertia and gravity, we arrive at

$$\frac{\partial \varepsilon}{\partial t} + \nabla(\varepsilon \mathbf{v}) = 0, \quad 0 = -\nabla p - n \langle \mathbf{f} \rangle, \quad \varepsilon = 1 - \rho. \tag{13}$$

For the force of particle interaction in a unit volume of the mixture with the gas in the system of randomly packed but not pulsating particles, we will take the expression ensuing from the analysis in [9], $\mathbf{u} = \mathbf{v} - \mathbf{w}$:

$$n\mathbf{f} = \frac{d_0}{a} \rho K(\rho) u \mathbf{u} - \rho \nabla p, \quad K = \frac{3\zeta}{8} \left(\frac{1-\rho}{1 - 1{,}17\rho^{2/3}} \right)^2. \tag{14}$$

Here, $1 - 1.17\rho^{2/3}$ is the minimal relative flow section in the particle system, whereas the resistance coefficient ξ monotonically and fairly slowly decreases with increasing Reynolds number for the particles, tending asymptotically to 0.5 [9, 10]. Instead of Eq. (14), we could also use some empirical representation, for instance, a two-term Ergan formula, applied traditionally in analyzing the behavior of dispersed systems. The second term in Eq. (14) represents a buoyancy force.

We now write the Langevin equation for pulsatory motion of a certain particle

$$m d\mathbf{w}'/dt = \mathbf{f}' + \mathbf{f}_c', \tag{15}$$

where $\mathbf{f}'$ is the fluctuation of the force of interaction with the gas in the presence of pulsations, in conformity with Eq. (14) equal to:

$$\mathbf{f}' = \frac{d_0}{a} \sigma \left[K u \mathbf{u}' + K(\mathbf{u}_0 \mathbf{u}') \mathbf{u} + \frac{dK}{d\rho} u \mathbf{u} \rho' \right] - \sigma \nabla p', \tag{16}$$

and $\mathbf{f}_c$ is the random force acting on the considered particle in collisions with other particles. In Eq. (16), $\mathbf{u}_0 = \mathbf{u}/u$ is the unit vector in the direction of the average relative gas velocity, and K and $dK/d\rho$ are predicted for the average volume concentration. By definition, $\mathbf{u}' = \mathbf{v}' - \mathbf{w}'$, i.e., the force $\mathbf{f}'$ contains a constituent linear with respect to $\mathbf{w}'$ and a constituent which does not depend on $\mathbf{w}'$. It is natural to identify the viscous dissipation with the work of the first constituent, the energy supply to pulsations with the work of the second constituent of the force $\mathbf{f}'$, and the dissipation at collisions with the work of the force $\mathbf{f}_c'$. Thus, on a unit volume basis

$$q = \frac{d_0}{a} \rho \left\{ K \left[\langle \mathbf{v}'\mathbf{w}' \rangle + \langle (\mathbf{u}_0' \mathbf{v}')(\mathbf{u}_0 \mathbf{w}') \rangle \right] \mathbf{u} + \right.$$

$$\left. + \frac{dK}{d\rho} u \langle \rho' \mathbf{w}' \rangle \mathbf{u} \right\} - \rho \langle \mathbf{w}' \nabla p' \rangle,$$

$$q_v = (d_0/a) \rho K \left[-\langle \mathbf{w}'^2 \rangle - \langle (\mathbf{u}_0 \mathbf{w}')^2 \rangle \right] u, \quad q_c = -n \langle \mathbf{f}_c' \mathbf{w}' \rangle.$$

Obviously, the properties of $\mathbf{f}_c'$ are governed entirely by the mixture concentration, the effective temperature θ, and by the physical characteristics. Moreover, taking into account $\langle \mathbf{w}'^2 \rangle - \langle (\mathbf{u}_0 \mathbf{w}')^2 \rangle = 2\theta/m$, we hence have

$$q_v = \alpha_v \theta, \quad \alpha_v = 2nK(u/a)(d_0/d_1), \quad q_c = q_c(\theta). \tag{17}$$

Let us examine the particle gas states analogous to the states of thermodynamic equilibrium of the molecular gas. In these states, all unknown variables (of course, except the gas pressure) are uniform. In such a state, from Eq. (3) or (15) we obtain

$$q_e - q_{v,e} - q_{c,e} = q_c - \alpha_v \theta_e - q_c(\theta_e), \tag{18}$$

where the supplementary subscript implies that a relevant quantity refers to the equilibrium state.

Taking into account linearity as to the pulsatory velocity of the particle, it is natural to adopt $q = \alpha\sqrt{\theta}$, considering in the first approximation that α is totally independent of θ. In this case, the indicated coefficient can be found from Eq. (18), which yields

$$q = \alpha \sqrt{\theta}, \quad \alpha = \alpha_v \sqrt{\theta_e} + q_c(\theta_e)/\sqrt{\theta_e}. \tag{19}$$

Hence and from Eq. (17) we further obtain

$$q - q_v - q_c = \alpha_v \sqrt{\theta}\,(\sqrt{\theta_e} - \sqrt{\theta}) + \sqrt{\theta}\left(\frac{q_c(\theta_e)}{\sqrt{\theta_e}} - \frac{q_c(\theta)}{\sqrt{\theta}}\right), \tag{20}$$

defining a previously unknown term in Eq. (3) for pulsatory energy transfer.

The quantity qc may be represented as a product of the collision frequency in a concentrated particle gas by the average energy loss at one collision. The multiplier is readily expressed as a function of ρ and θ from the Enskog theory, while the multiplicand depends additionally on the viscoelastic properties of the particle material and on the viscosity of the gas surrounding the particles. It may in principle be determined through the same methods as those used in [21] to study liquefaction of dispersed ferromagnetics by a variable magnetic field. For simplicity, we totally disregard the energy loss at collisions, setting $q_c = 0$.

Thus, the problem of defining the quantities q and q_v reduces, in fact, to a calculation of the equilibrium temperature θ_e as a function of unknown average variables and physical parameters. To approximately assess this quantity, we will make use of the considerations from Eq. (11), whence the energy of particle pulsations in the real fluidized bed coincides, in the first approximation, with a similar energy in a certain fictitious layer of the same particles and at the same values of ρ and u, with no collisions.

Denoting, in order to avoid misunderstanding, the pulsations of phase velocities and of gas pressure in a fictitious layer by capitalized symbols, we obtain from Eqs. (13) and (15) for $\mathbf{f}_c' = 0$ the equations

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\nabla\right)\rho' = \varepsilon\nabla\mathbf{V}', \quad 0 = -\nabla\mathbf{P}' - n\mathbf{F}', \quad \rho d_1 \frac{\partial \mathbf{W}'}{\partial t} = n\mathbf{F}', \tag{21}$$

here $\mathbf{F}'$is expressed in terms of ρ', $\mathbf{P}'$, $\mathbf{V}'$, and $\mathbf{W}'$ in the same way as $\mathbf{f}'$ from Eq. (16) is expressed in terms of ρ', p', v', and $\mathbf{w}'$. Equations (21) are written in a coordinate system, where the particles are on the average immovable.

In solving Eq. (21) we utilized, just as [8], a correlation theory of stationary random processes [22], according to which any random function R′ with a zero average and a stationary root-mean-square value is presented in the form of Fourier–Stieltjes type integral with a random measure dZ_R:

$$R'(t, \mathbf{r}) = \int \exp[i(\omega t + \mathbf{kr})]\, dZ_R,$$

and the average of a multiplication of two stationary random functions is defined as the integral, over the entire frequency axis and wave space, of an appropriate spectral density:

$$\langle R'S' \rangle = \int_{-\infty}^{+\infty} d\omega \int d\mathbf{k}\Psi_{R,S}(\omega, \mathbf{k}),$$

$$\Psi_{R,S}(\omega, \mathbf{k}) = \lim_{\substack{d\omega \to 0 \\ d\mathbf{k} \to 0}} \frac{\langle dZ_R\, dZ_S^* \rangle}{d\omega d\mathbf{k}},$$

where the asterisk denotes an operation of complex conjugation.

It is easy to obtain the linear algebraic equations for random measures from Eqs. (21), permitting all measures to be expressed as the quantities proportional to dZ_ρ. With a view to Eq. (16), these equations are as follows:

$$\begin{gathered}(\omega + \mathbf{uk})\, dZ_\rho = \varepsilon\mathbf{k} d\mathbf{Z}_{\mathbf{v}}, \quad \rho d_1 \omega d\mathbf{Z}_w = -\mathbf{k} dZ_P, \\ (d_0/a)\,\rho\Big[Ku(d\mathbf{Z}_V - d\mathbf{Z}_W) + K(\mathbf{u}_0(d\mathbf{Z}_V - d\mathbf{Z}_W))\,\mathbf{u} + \\ + \frac{dK}{d\rho}\cdot u\cdot \mathbf{u} dZ_\rho\Big] = -i\varepsilon\mathbf{k} dZ_P.\end{gathered} \tag{22}$$

Solving Eq. (22) and subsequently calculating the spectral measure of the i-th component of the particle pulsatory velocity result in

$$\Psi_{w_j w_j}(\omega,\ \mathrm{k}) = \frac{k_j^2}{k^4}\left[1 + \frac{\omega^2 \varepsilon a d_1}{K u d_0}\left(1 - \frac{1}{2}\,\frac{k_1^2}{k_2^2}\right)\right]^{-1} \times \\ \times \left[\frac{\omega}{\varepsilon} + u k_1\left(\frac{1}{\varepsilon} + \frac{1}{2}\,\frac{d \ln K}{d\rho}\right)\right]^2 \Psi_{\rho,\rho}(\omega,\ \mathrm{k}),\quad k_1 = \mathrm{u}_0 \mathrm{k}. \tag{23}$$

To close this relation, we use an expression for the spectral density of random concentration fluctuations of monodispersed systems, ensuing from the theory in [23]:

$$\Psi_{\rho,\rho}(\omega,\ \mathrm{k}) = \frac{3}{4\pi}\,\frac{\rho^2}{k_0^3}\left(1 - \frac{\rho}{\rho_*}\right)\frac{k^2 D/\pi}{\omega^2 + (k^2/D)^2},\quad k_0^3 = 9\pi\rho/2a^3. \tag{24}$$

Here D is the self-diffusion coefficient of particles determined in Eq. (12). Equations (23) and (24) allow us to obtain, by general rule, $\langle w'^2\rangle$, and, consequently, the desired temperature θ_e of the equilibrium state characterized by the specified ρ and u.

The self-diffusion coefficient in a concentrated system is very small. This implies, in particular, that the values of ω, making a major contribution to the integral along the frequency axis, are small as well. Therefore, solely for the purpose of simplifying the calculations, we can set $\omega = 0$ in the first approximation in Eq. (23). Then, after a simple calculation we get

$$\theta_e = \frac{\rho^2}{9}\left(1 - \frac{\rho}{\rho_*}\right)\left(\frac{1}{\varepsilon} + \frac{1}{2}\,\frac{d \ln K}{d\rho}\right)^2 m u^2. \tag{25}$$

Equilibrium states are homogeneous. Eliminating the pressure gradient and taking account of Eq. (14), from Eq. (2) and from the second equation (13) we can obtain for a homogeneous system:

$$u = \left(\frac{d_1}{d_0}\,\frac{\varepsilon a g}{K}\right)^{1/2} = \frac{1 - 1{,}17\rho^{2/3}}{\sqrt{1-\rho}}\,u^\circ,\quad u^\circ = \left(\frac{8 a g d_1}{3\zeta d_0}\right)^{1/2}, \tag{26}$$

where u_0 has the meaning of a floatation velocity. Equations (25) and (26) yield

$$\theta_e = \frac{\rho^2}{9}\left(1 - \frac{\rho}{\rho_*}\right)\left(\frac{1}{\varepsilon} + \frac{1}{2}\,\frac{d \ln K}{d\rho}\right)^2 \left(\frac{1 - 1{,}17\rho^{2/3}}{\sqrt{1-\rho}}\right)^2 m (u^\circ)^2. \tag{27}$$

The dependence of the dimensionless effective temperature on the concentration is given in Fig. 3. This dependence has a maximum, analogous to maxima in the theory of studies [8-11].

An equilibrium pressure of the particle gas can be presented on the basis of Eqs. (6) and (27) as

$$P_e = \frac{\rho^3}{9}\left(1 - \frac{\rho}{\rho_*}\right)\left(\frac{1}{\varepsilon} + \frac{1}{2}\,\frac{d \ln K}{d\rho}\right)^2 \left(\frac{1 - 1{,}17\rho^{2/3}}{\sqrt{1-\rho}}\right)^2 G(\rho)\, d_1 (u^\circ)^2. \tag{28}$$

Figure 4 plots the dimensionless pressure as a function of the concentration for $G(\rho)$ from Eq. (9). Importantly, a maximum disappears on this dependence. This means that the condition $dP_e/d\rho > 0$ of thermodynamic stability for the considered particle gas is fulfilled over the entire concentration range, so that a stability disturbance of the homogeneous state can result only from the occurrence of instability of purely hydrodynamic origin. In this connection, we must mention a basic defect of the theory in [9, 20, 24], which, unfortunately, has also penetrated into the literature (see, for example, [25, 26]). In the indicated studies, a region of relatively low concentrations with $dP_e/d\rho > 0$ is identified with vaporlike states of the system of suspended particles, whereas a region of high concentrations with $dP_e/d\rho < 0$ is likened to liquidlike states of this system. In this case, a critical value of the concentration, providing a maximum to the equilibrium particle pressure, is treated as corresponding to a certain analog of the liquid–vapor phase transitions in molecular systems, whereas precisely the liquidlike states are assumed stable.

Obviously, the above inferences stem from misunderstanding. Particularly, a random compaction of the layer for $dP_e/d\rho < 0$ will persist at the expense of the pressure of the surrounding particle gas, which is higher than such a pressure inside the packing. Similarly, the spontaneous rarefaction will continue expanding, since the pressure therein is higher than the ambient pressure. As a result, the layer must break down into close packing regions alternated with regions, where the particle concentration is equal to a critical value, complying with the maximum of P_e. Clearly, such a process is utterly analogous with the known process of spinoidal disintegration under the conditions of absolute thermodynamic instability of the molecular systems.

It should be stressed that the average force $n\langle \mathbf{f} \rangle$, entering into the equation of momentum conservation of the dispersed phase (2), for the real layer with pulsating particles somewhat differs from the force $n\mathbf{f}$ from Eq. (14) for the layer of immovable particles of the same concentration. This effect was tackled previously in [8]. In the first approximation it can, apparently, be disregarded, which exactly was done when calculating **u** in Eq. (26).

Thus, for describing a macroscopic motion of the monodispersed system with large particles, we have a system of three scalar and two vector equations in Eqs. (1)-(3) and (13), with the scalars ρ, p, and θ, and the vectors **u** and **w** serving as their unknown variables. This system is closed by rheological relations for the stress tensor and for the pulsatory energy flux in Eqs. (4) and (5), whose coefficients are expressed in terms of the unknown variables and physical parameters proper, in accordance with Eqs. (6) and (9)-(10). The force of interphase interaction $n(\mathbf{f})$ and the energy influx $q - q_v$ (in the case with $q_c = 0$) are described approximately by relations (14) and (20), the coefficient α_v being defined in Eq. (17) and the equilibrium temperature of particle gas θ_e in Eq. (27).

The present work is not aimed at solving any specific problems regarding the behavior of fluidized beds. However, pursuing purely illustrative purposes and bearing in mind future applications, we will discuss the equations describing stationary particle distributions along a height of the fluidized bed. In the laboratory coordinate system, $\mathbf{w} = 0$. Eliminating the gas pressure gradient from Eqs. (2), (3), and (13), we arrive at a system of two equations determining the profiles $\rho(x)$ and $\theta(x)$, where x is the vertical coordinate running upwards:

$$\begin{gathered}\frac{d}{dx}[\rho G(\rho)\theta] = -m\rho\left[g - \frac{K(\rho)}{\varepsilon^3}\frac{d_0}{d_1}\frac{U^2}{a}\right],\\ \frac{d}{dx}\left[\varphi(\rho)\frac{d}{dx}(\theta^{3/2})\right] = -\frac{768}{75}\sqrt{\frac{3}{4}}\left(\sqrt{\frac{d_1}{a}}U\right)\times\\ \times\frac{\rho K(\rho)}{\varepsilon}\sqrt{\theta}\left(\sqrt{\theta_e} - \sqrt{\theta}\right),\end{gathered} \tag{29}$$

with the function $\varphi = \lambda/\lambda^0$ defined in Eq. (10), θ_e in Eq. (27), and $U = \varepsilon u$ representing the volume gas flow (i.e., the velocity on the basis of a void section of the device).

In analyzing the particle distribution over the upper part of the fluidized bed, the transfer of the pulsatory particle energy by the pulsations proper may mostly be ignored. This corresponds to the use of an approximation analogous to the Euler hydrodynamic approximation. In this case, the second equation (29) gives $\theta = \theta_e$, whereas the first equation, after a simple manipulation with allowance for Eq. (27), takes the form

$$\begin{gathered}\frac{d}{d\rho}\left\{\rho^3\left(1 - \frac{\rho}{\rho_*}\right)\left(1 + \frac{\varepsilon}{2}\frac{d\ln K}{d\rho}\right)^2\left(\frac{1 - 1{,}17\rho^{2/3}}{\sqrt{\varepsilon}}\right)^2 G(\rho)\right\}\frac{d\rho}{dx} = \\ = -\frac{27\zeta\rho}{8a}\frac{d_0}{d_1}\left[1 - \frac{U^2}{\varepsilon(1 - 1{,}17\rho^{2/3})^2(u^{\circ})^2}\right].\end{gathered} \tag{30}$$

The distributions are stable which decrease with height. Such distributions exist in a certain concentration interval $(0, \rho_1)$, where the quantity in square brackets on the right side of Eq. (30) is positive. Considering $\rho_1 = 0$ and $\rho_1 = \rho_*$, we hence obtain the known condition of fluidization

$$\sqrt{\varepsilon_*}\,(1 - 1{,}17\rho_*^{2/3}) < U/u^{\circ} < 1. \qquad (31)$$

When the left inequality is violated, the granular layer is in an immobile state, while with violation of the right inequality the particles are carried away from the device by a flow of the air or another gas. Integral curves of Eq. (30) are easy to examine similarly to the investigation of integral curves of the equation in [25], following from the inadequate theory in [9, 10].

A concentration profile in the near-grid zone of the fluidized bed is dependent on a uniform gas distribution over the grid. With the distribution fairly smooth, the near-grid zone does not, actually, differ in this respect from the layer volume far off the grid (where $\rho \approx \rho_1$, and ρ_1 is the root of the equation $\sqrt{\varepsilon^1}(1 - 1.17\rho^{2/3}) = U/u^0$). However, if the gas distribution is essentially nonuniform, then, as a result of the interaction with nonuniformities, the particles attain the temperature θ_0 exceeding the equilibrium value θ_e at $\rho = \rho_1$. In this case, the pulsatory energy is transferred from the grid deep into the layer, whereas a region of reduced concentration forms near the grid, which is verified by numerous experiments. The profiles $\theta(x)$ and $\rho(x)$ are described by equations of the form (29), satisfying the Navier–Stokes approximation.

NOTATION

a, particle radius; D, self-diffusion coefficient; d_1, density of the particle material; **E**, tensor of strain rates; **f**, force acting on the particle from the side of the gas; $\mathbf{f}_c'$, force caused by collisions; G, function introduced in Eq. (6); g, acceleration due to gravity; **I**, unit tensor; K, function introduced in Eq. (14); **k**, wave vector; m, particle mass, n, numerical concentration of the particles; **P**, P, stress tensor and pressure of the particles; p, gas pressure; **Q**, energy flux of pulsations; q, q_v, q_c, energy supply to pulsations, and dissipation in the gas and due to collisions, respectively; U, volume flow (filtration rate) of the gas; u, relative velocity of the gas; $\mathbf{u}_0 = \mathrm{u/u}$; **v**, **w**, gas and particle velocities, respectively; y, parameter defined in Eq. (10); x, vertical coordinate; dZ, random measure; α, α_v, coefficients in Eqs. (17) and (19), respectively; ε, porosity; ζ, resistance coefficient of the particle; θ, effective temperature of the particle gas; μ, particle gas viscosity; λ, transfer coefficient of pulsatory energy; ρ, ρ_*, volume concentration of the dispersed phase and its value in a state of close packing; χ, Enskog factor; ψ, spectral density; ω, frequency. Superscripts: ′, pulsations of variables; °, quantities for the rarefied particle gas. Subscripts: e, parameters of equilibrium states.

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